Modes of nonlinear acoustic transparency in the strained paramagnetic crystal

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The propagation of transverse-longitudinal acoustic pulses through a strained cubic crystal containing resonant paramagnetic impurities with effective spin $S=1$ is investigated. It is supposed that the pulses propagate under arbitrary angle with respect to the direction of the external static deformation parallel to the fourth-order symmetry axis. In this geometry, both the transverse and longitudinal components of the acoustic field have high-frequency and zero-frequency spectral components. We show that a pulse can propagate in modes different from the acoustic self-induced transparency. In particular, a pulse propagating in the mode of an acoustic self-induced supertransparency substantially changes the populations of the spin sublevels, but its group velocity remains almost equal to the linear velocity of the sound. If a pulse propagates in the acoustic extraordinary transparency mode, then its group velocity is substantially lower while the sublevel populations remain virtually invariant. Also, the modes of propagation under conditions of weakly excited spin transitions and large detuning of the pulse high-frequency components are identified.

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I. INTRODUCTION

Historically, the coherent optical effects found their acoustic analogs after a while. Such a correspondence is easily traced, for instance, in the case of the self-induced transparency (SIT) phenomenon. Its discovery in 1967 [1] and theoretical explanation in 1969 [2] stimulated a search for the acoustic analog. The experimental observation at the liquid-helium temperature of the acoustic self-induced transparency (ASIT) of the longitudinal hypersound in cubic crystal MgO containing paramagnetic ions Fe^{2+} and Ni^{2+} was carried out in 1970 by Shiren [3], who gave also a theoretical consideration of this effect. Independently, ASIT was studied theoretically for transverse hypersound in a system of paramagnetic impurities with effective spin $S=1/2$ [4]. Soon after that, this effect was successfully revealed in a crystal $LiNbO_3$ alloyed by ions Fe^{2+} [5].

Despite the existence of the optical-acoustic analogies mentioned above, acoustic coherent effects do not appear as exact copies of optical ones. Obviously, quantitative differences connected with the great distinction of light and sound velocities or frequencies of electric-dipole and spin-phonon transitions take place. It should be stressed that qualitative differences exist there. Thus, the optical wave is especially transverse whereas the acoustic one has longitudinal-transverse structure.

The interaction between transverse and longitudinal components of the acoustic pulse in a crystal without anharmonicity of the lattice modes happens due to the presence of paramagnetic impurities. Its influence on the dynamics of the components is inessential if the linear velocities of acoustic waves differ significantly, as it takes place in most solids.

However, the linear velocities of transverse and longitudinal sound are close in the ion crystals of halogenide of alkaline metals [6]. Since the components of the strain field interact effectively in this case, new features of their dynamics have to be expected. Indeed, a new mode of the resonant acoustic transparency has been revealed theoretically in Ref. [7]. The propagation of transverse-longitudinal elastic pulses in this mode is accompanied by trapping of the population of the spin sublevels. Nevertheless, the deceleration in the velocity of the pulses is comparable to that in the case of ASIT of especially longitudinal or transverse hypersound [3,4].

The theoretical considerations of the nonlinear dynamics of transverse-longitudinal acoustic pulses were fulfilled basically for paramagnetic ions with effective spin $S=1/2$ [7–9]. At the same time, it is well known that the strongest interaction with crystal lattice oscillations takes place in the case $S=1$ [6]. Since the spin-phonon interaction is greater here for two or three orders than for $S=1/2$, investigation of the acoustic transparency of transverse-longitudinal pulses in a system of paramagnetic impurities with effective spin $S=1$ is more preferable from the point of view of experimental testing.

The resonant modes of propagation of the transverse-longitudinal hypersound in a direction parallel to the external magnetic field and an axis of symmetry of the fourth order of the paramagnetic crystal with $S=1$ have been studied in [10]. It was shown that these modes coincide with the ones discussed in Ref. [7]. The distinctive feature of the geometry considered there is that the roles of transverse and longitudinal components of the acoustic pulses are strictly various: the former component excites the spin-phonon transitions in the Zeeman triplet, and the latter one causes dynamic shifts of the frequencies of these transitions.

Recently, it has been shown in the case of especially longitudinal acoustic pulses propagating under arbitrary angle with respect to an axis of symmetry of the paramagnetic crystal that both functions can be carried out by a single

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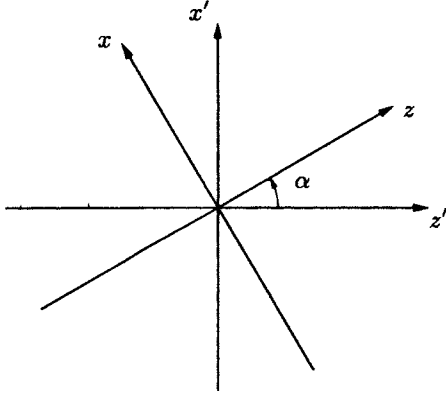
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FIG. 1. Scheme of the coordinate axes.

component of the strain field [11]. In this connection, study of the nonlinear modes of the acoustic transparency for the transverse-longitudinal pulses that propagate in a system of effective spins $S=1$ under arbitrary direction to the symmetry axis is of great interest. The present report is devoted to allocating these modes. Following Ref. [11], we suppose here that the external magnetic field is absent and that the splitting of the energy sublevels is created by static deformation of the paramagnetic crystal. For this reason, the degeneration of the spin states is removed incompletely in contrast to the usual ASIT.

The paper is organized as follows. In Sec. II, the system of material and wave equations describing the interaction in the strained crystal of the transverse-longitudinal elastic pulses with the paramagnetic impurities having effective spin $S=1$ is derived. We use the semi classical approach; i.e., the dynamics of the acoustic fields is assumed to obey the classical Hamilton equations for a continuous medium, but the spin subsystem is treated as a quantum object. The order of the derivatives in the wave equations on the strain field components is reduced with the help of the unidirectional propagation (UP) approximation. In Sec. III, the approximation of slowly varying envelopes (SVE) is applied to the particular case of the system obtained in the previous section. It is shown that the evolution of the acoustic pulses in the paramagnetic crystal is governed by well-studied equations of long- and short-wave coupling (LSWC) [12,13]. The pulse solutions decreasing exponentially and rationally of the LSWC equations are presented in Sec. IV. These solutions are exploited to classify the modes of the acoustic transparency of transverse-longitudinal pulses in the deformed paramagnetic crystal.

II. BASIC MODEL

Consider a cubic crystal containing paramagnetic impurities with effective spin $S=1$. Let a transverse-longitudinal acoustic pulse propagate in the crystal along the z axis directed under arbitrary angle α with respect to one of its axes of symmetry of the fourth order (z' axis; see Fig. 1). Also, we assume that an external field of static and uniform deformation employing the crystal is parallel to the z' axis.

The Hamiltonian $\hat{H}$ of the spin-elastic interaction that arises in the most general case [14] is a function of the bi-

linear combination of spin operators $\hat{S}_j$ ($j=x',y,z'$):

$$\hat{H} = f(\hat{S}_i Q_{ij} \hat{S}_j). \tag{1}$$

Coefficients Q_{ij} in this formula are supposed to depend on the components of the strain tensor:

$$\mathcal{E}_{ml} = \frac{1}{2}\left(\frac{\partial U_m}{\partial x_l} + \frac{\partial U_l}{\partial x_m}\right)$$

($m,l=x',y,z'$; U_m are the components of the displacement vector $\mathbf{U}$). In a representation of the eigenfunctions of operator $\hat{S}_{z'}$, the spin matrices take the form [6]

$$\hat{S}_{x'} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad \hat{S}_y = \frac{i}{\sqrt{2}}\begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix},$$

$$\hat{S}_{z'} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}. \tag{2}$$

The coefficients of the bilinear combination satisfy the condition $Q_{ij}=\delta_{ij}$ if the external deformation of the crystal is absent. Expanding these coefficients and function f in the power series on $\mathcal{E}_{ml}$ and retaining the first-order terms, we obtain

$$f = f(\hat{S}^2) + f'(\hat{S}^2)\left(\frac{\partial Q_{ij}}{\partial \mathcal{E}_{ml}}\right)_0 \hat{S}_i \hat{S}_j \mathcal{E}_{ml},$$

where the subscript "0" means that the values of derivatives correspond to nondeformed crystal. The first term depending on the Casimir operator $\hat{S}^2 = S(S+1) = 2$ is omitted in what follows as a constant addition to $\hat{H}$. Then, the Hamiltonian (1) is expressed through the components of the tensor of spin-elastic interaction,

$$G_{ijlm} = f'(\hat{S}^2)\left(\frac{\partial Q_{ij}}{\partial \mathcal{E}_{ml}}\right)_0,$$

given as

$$\hat{H} = \frac{1}{2}G_{ijlm}\mathcal{E}_{ml}(\hat{S}_i\hat{S}_j + \hat{S}_j\hat{S}_i).$$

In the case of the geometry we consider, the last equation is rewritten in the following manner:

$$\hat{H} = \hat{H}_0 + \hat{V},$$

where $\hat{H}_0$ is the Hamiltonian of the effective spin in the field $\mathcal{E}_{z'z'}^{(0)}$ of the external deformation

$$\hat{H}_0 = G_\parallel \mathcal{E}_{z'z'}^{(0)}\hat{S}_{z'}^2 = \hbar\,\omega_0\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{3}$$

and $\hat{V}$ is the Hamiltonian of the interaction between the effective spin and strain field of the acoustic pulse propagating through the crystal [14]:

$$\hat{V} = G_{\parallel} \mathcal{E}_{zz} \hat{S}_z^2 + \frac{G_{\perp}}{2} [\mathcal{E}_{xz}(\hat{S}_x \hat{S}_z + \hat{S}_z \hat{S}_x) + \mathcal{E}_{yz}(\hat{S}_y \hat{S}_z + \hat{S}_z \hat{S}_y)].$$

$$(4)$$

Here we use the notation $G_{\parallel} = G_{z'z'z'z'} = G_{x'x'x'x'} = G_{yyyy}$, $\omega_0 = G_{\parallel} \mathcal{E}_{z'z'}^{(0)} / \hbar$ ($\hbar$ is the Planck constant), and $G_{\perp} = G_{x'x'z'z'} = G_{x'x'yy} = G_{yyz'z'}$. The operators $\hat{S}_x$ and $\hat{S}_z$ are connected with $\hat{S}_{x'}$ and $\hat{S}_{z'}$ by means of the transformation of rotation [3,15]:

$$\hat{S}_x = \hat{S}_{x'} \cos \alpha - \hat{S}_{z'} \sin \alpha,$$

$$\hat{S}_z = \hat{S}_{z'} \cos \alpha + \hat{S}_{x'} \sin \alpha. \qquad (5)$$

For consideration of the dynamics of the effective spins and the acoustic pulse to be self-consistent, we introduce the Hamiltonian of the elastic field:

$$H_a = \frac{1}{2} \int \left\{ \frac{p_x^2 + p_y^2 + p_z^2}{\rho} + \rho a_{\parallel}^2 \left(\frac{\partial U_z}{\partial z} \right)^2 \right.$$
$$\left. + \rho a_{\perp}^2 \left[\left(\frac{\partial U_x}{\partial z} \right)^2 + \left(\frac{\partial U_y}{\partial z} \right)^2 \right] \right\} d\mathbf{r}, \qquad (6)$$

where ρ is the mean crystal density; p_x, p_y, and p_z are the components of the momentum density of the local displacement of the crystal; $a_{\parallel}$ and $a_{\perp}$ are linear velocities of the longitudinal and transversal hypersounds, respectively. We assume in Eq. (6) that all dynamical displacements U_x, U_y, and U_z depend on the variables z and t only. The integral is taken over the crystal volume.

In accordance with the general scheme of the semi classical approach [14,16], we describe the evolution of the effective spin by the equation on the density matrix $\hat{\rho}$:

$$i \hbar \frac{\partial \hat{\rho}}{\partial t} = [\hat{H}_0 + \hat{V}, \hat{\rho}]. \qquad (7)$$

At the same time, the dynamics of the acoustic field obeys the classical Hamilton equations for a continuous medium:

$$\frac{\partial \mathbf{p}}{\partial t} = -\frac{\delta}{\delta \mathbf{U}} (H_a + \langle \hat{\bar{V}} \rangle),$$

$$\frac{\partial \mathbf{U}}{\partial t} = \frac{\delta}{\delta \mathbf{p}} (H_a + \langle \hat{\bar{V}} \rangle), \qquad (8)$$

where

$$\langle \hat{\bar{V}} \rangle = \int n \langle \hat{V} \rangle d\mathbf{r}$$

and n is the concentration of the paramagnetic impurities; $\langle \hat{V} \rangle = \mathrm{Tr}(\hat{\rho} \hat{V})$ is the quantum average of $\hat{V}$.

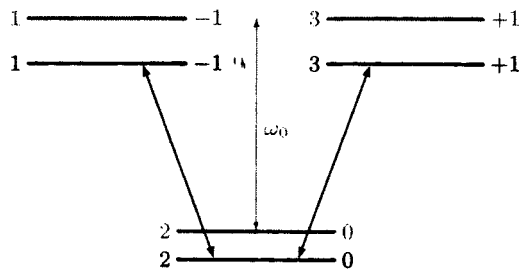Let the density matrix be represented in the next form

FIG. 2. Splitting of spin sublevels in the field of the static deformation (gray color), their position in the presence of the acoustic pulse, and quantum transitions excited. The number of the level and the corresponding value of projection $S_{z'}$ of effective spin on the axis of the external deformation are indicated from the left and right, respectively.

$$\hat{\rho} = \begin{pmatrix} \rho_{33} & \rho_{32} & \rho_{31} \\ \rho_{23} & \rho_{22} & \rho_{21} \\ \rho_{13} & \rho_{12} & \rho_{11} \end{pmatrix}. \qquad (9)$$

Equations (2), (4), and (5) give us the following expression for the Hamiltonian of the spin-phonon interaction:

$$\hat{V} = \begin{pmatrix} V_{11} & -V_{12}^* & V_{13}^* \\ -V_{12} & V_{22} & V_{12}^* \\ V_{13} & V_{12} & V_{11} \end{pmatrix}, \qquad (10)$$

where

$$V_{11} = \frac{G_{\parallel}}{2} (1 + \cos^2 \alpha) \mathcal{E}_{zz} - \frac{G_{\perp}}{2} \sin 2\alpha \mathcal{E}_{xz},$$

$$V_{22} = G_{\parallel} \sin^2 \alpha \mathcal{E}_{zz} + G_{\perp} \sin 2\alpha \mathcal{E}_{xz},$$

$$V_{12} = -\frac{G_{\parallel} \sin 2\alpha}{2\sqrt{2}} \mathcal{E}_{zz} - \frac{G_{\perp}}{\sqrt{2}} (\cos 2\alpha \mathcal{E}_{xz} + i \cos \alpha \mathcal{E}_{yz}),$$

$$V_{13} = \frac{G_{\parallel}}{2} \sin^2 \alpha \mathcal{E}_{zz} + G_{\perp} (\sin 2\alpha \mathcal{E}_{xz}/2 + i \sin \alpha \mathcal{E}_{yz}).$$

The physical mechanism of the spin-elastic interaction in the case considered is the Van Vleck mechanism [6]. The strain fields modulate the intracrystalline electric field in the location of the paramagnetic ions. Due to the quadrupole Stark effect, the static gradient of the electric field causes a splitting of the quantum sublevels of effective spin $S=1$ that occur to be degenerate on the absolute value of its projection $S_{z'}$ [see Eq. (3) and Fig. 2]. The components of the acoustic pulse excite the electro quadrupole transitions between these sublevels and, as follows from the expressions for V_{11} and V_{22}, shift dynamically the transition frequency.

Using Eqs. (6) and (8)–(10), we deduce the wave equations on the strain field components:

$$\frac{\partial^2 \mathcal{E}_{zz}}{\partial t^2} - a_\parallel^2 \frac{\partial^2 \mathcal{E}_{zz}}{\partial z^2} = \frac{nG_\parallel}{2\rho} \frac{\partial^2}{\partial z^2} \left[(3\sin^2\alpha - 2)\rho_{22} + \frac{\sin 2\alpha}{\sqrt{2}}(\rho_{23} \right.$$

$$\left. + \rho_{32} - \rho_{12} - \rho_{21}) + \sin^2\alpha(\rho_{13} + \rho_{31}) \right],$$

$$(11)$$

$$\frac{\partial^2 \mathcal{E}_{xz}}{\partial t^2} - a_\perp^2 \frac{\partial^2 \mathcal{E}_{xz}}{\partial z^2} = \frac{nG_\perp}{4\rho} \frac{\partial^2}{\partial z^2} [3\sin 2\alpha\rho_{22} + \sqrt{2}\cos 2\alpha(\rho_{23}$$

$$+ \rho_{32} - \rho_{12} - \rho_{21}) + \sin 2\alpha(\rho_{13} + \rho_{31})],$$

$$(12)$$

$$\frac{\partial^2 \mathcal{E}_{yz}}{\partial t^2} - a_\perp^2 \frac{\partial^2 \mathcal{E}_{yz}}{\partial z^2} = i\frac{nG_\perp}{2\rho} \frac{\partial^2}{\partial z^2} \left[\sin\alpha(\rho_{31} - \rho_{13}) \right.$$

$$\left. + \frac{\cos\alpha}{\sqrt{2}}(\rho_{12} + \rho_{32} - \rho_{21} - \rho_{23}) \right].$$

$$(13)$$

Equations (7) and (11)–(13) form a self-consistent system describing the nonlinear dynamics of transverse-longitudinal acoustic pulses in the deformed crystal containing the paramagnetic impurities with effective spin $S=1$. Since its analysis is very complicated in the general case, we restrict our subsequent considerations to a specific model.

Let us assume that the linear velocities of both acoustic waves are equal:

$$a_\parallel = a_\perp = a.$$

This restriction is rather artificial, since $a_\parallel > a_\perp$ in solids. As noted in the previous section, it is fulfilled best of all in the ion crystals of halogenide of alkaline metals [6]. One of the representatives of such the crystals is, for instance, NaBr [17]. Using typical experimental parameters of medium and acoustic pulses, we show at the end of Sec. IV that strict observance of this condition is not very important from a physical point of view. The case of acoustic pulse propagation through deformed paramagnetic crystal with effective spin $S=1$, in which the linear velocities of the components differ essentially, has been studied in Ref. [11].

As far as the linear velocities coincide, we are able to simplify Eqs. (11)–(13) by reducing them in the order of the derivative with the help of the UP approximation [18]. Indeed, suppose that the concentration of the paramagnetic impurities in the crystal is small,

$$\eta = \frac{n(G_\perp^2 + G_\parallel^2)}{\hbar \omega_0 \rho a^2} \ll 1,$$

and introduce new independent variables

$$\tau = t - \frac{z}{a}, \quad \zeta = \eta z,$$

which are usually referred to as the retarded time and slow coordinate, respectively. Obviously, we have

$$\frac{\partial}{\partial t} = \frac{\partial}{\partial \tau}, \quad \frac{\partial}{\partial z} = -\frac{1}{a}\frac{\partial}{\partial \tau} + \eta\frac{\partial}{\partial \zeta}.$$

Then, neglecting terms proportional to η^2, we approximately write

$$\frac{\partial^2}{\partial z^2} \approx \frac{1}{a^2}\frac{\partial^2}{\partial \tau^2} - 2\frac{\eta}{a}\frac{\partial^2}{\partial \tau \partial \zeta}$$

on the left-hand side of Eqs. (11)–(13) and

$$\frac{\partial^2}{\partial z^2} \approx \frac{1}{a^2}\frac{\partial^2}{\partial \tau^2}$$

on the right hand side. Integration of τ of the wave equations obtained in this way in the terms of new variables gives, after returning to the initial ones,

$$\frac{\partial \mathcal{E}_{zz}}{\partial z} + \frac{1}{a}\frac{\partial \mathcal{E}_{zz}}{\partial t} = \frac{nG_\parallel}{4\sqrt{2}\,\hbar\,\rho a^3}\{i(\rho_{12} - \rho_{21} + \rho_{23} - \rho_{32})$$

$$\times(\hbar\omega_0\sin 2\alpha - 2G_\perp\mathcal{E}_{xz})$$

$$+ 2G_\perp[\sqrt{2}\sin\alpha(\rho_{33} - \rho_{11}) - \cos\alpha(\rho_{12} + \rho_{21}$$

$$+ \rho_{23} + \rho_{32})]\mathcal{E}_{yz}\},\qquad (14)$$

$$\frac{\partial \mathcal{E}_{xz}}{\partial z} + \frac{1}{a}\frac{\partial \mathcal{E}_{xz}}{\partial t} = \frac{nG_\perp}{4\sqrt{2}\,\hbar\,\rho a^3}\{i(\rho_{12} - \rho_{21} + \rho_{23} - \rho_{32})$$

$$\times(\hbar\omega_0\cos 2\alpha + G_\parallel\mathcal{E}_{zz}) + G_\perp[\sqrt{2}\cos\alpha(\rho_{33}$$

$$- \rho_{11}) + \sin\alpha(\rho_{12} + \rho_{21} + \rho_{23} + \rho_{32})]\mathcal{E}_{yz}\},$$

$$(15)$$

$$\frac{\partial \mathcal{E}_{yz}}{\partial z} + \frac{1}{a}\frac{\partial \mathcal{E}_{yz}}{\partial t} = \frac{nG_\perp}{4\sqrt{2}\,\hbar\,\rho a^3}[\sqrt{2}(\rho_{11} - \rho_{33})(G_\perp\cos\alpha\mathcal{E}_{xz}$$

$$+ G_\parallel\sin\alpha\mathcal{E}_{zz}) + (\rho_{12} + \rho_{21} + \rho_{23} + \rho_{32})$$

$$\times(\hbar\omega_0\cos\alpha - G_\perp\sin\alpha\mathcal{E}_{xz} + G_\parallel\cos\alpha\mathcal{E}_{zz})].$$

$$(16)$$

It is assumed hereinafter that the components of the strain field and the density matrix elements satisfy the following conditions at $t \to -\infty$:

$$\mathcal{E}_{xz,yz,zz} \to 0, \quad \frac{\partial \mathcal{E}_{xz,yz,zz}}{\partial t} \to 0, \quad \rho_{jj} \to W_j, \quad \rho_{jk} \to 0$$

$(j,k=1,2,3; \ k \neq j)$. These conditions correspond to the pulsed mode of propagation of sound in a crystal. Note that the evolution equation (7) of the density matrix is used to exclude the time derivatives of its elements on the right-hand side of Eqs. (14)–(16).

The equations we came to are yet complicated for the rigorous analysis. Fortunately, they can be reduced in the particular case to a system describing the interaction of transverse-longitudinal acoustic pulses with two-level quantum particles. The investigation of this system will be carried out in the next sections.

III. SYSTEM OF LSWC EQUATIONS

Let the populations of the quantum states with $S_{z'} = \pm 1$ be equal:

$$\rho_{33} = \rho_{11}. \qquad (17)$$

This assumption is compatible with Eqs. (7) and (14)–(16) if the following conditions are imposed:

$$\rho_{31} = \rho_{13}, \quad \rho_{32} = -\rho_{12}, \quad \mathcal{E}_{yz} = 0. \qquad (18)$$

It is checked immediately that conditions (18) are sufficient to keep Eq. (17). Also, they give us relation

$$\rho_{13} = W_1 - \rho_{11}. \qquad (19)$$

Taking into account Eqs. (17)–(19), we rewrite Eqs. (7) and (14)–(16) in the next form:

$$\frac{\partial W}{\partial t} = iE_\perp(\sigma - \sigma^*), \qquad (20)$$

$$\frac{\partial \sigma}{\partial t} = i(\omega_0 + 2E_\parallel)\sigma + 2iE_\perp W, \qquad (21)$$

$$\frac{\partial E_\perp}{\partial z} + \frac{1}{a}\frac{\partial E_\perp}{\partial t} = -i\frac{nA}{4\,\hbar\,\rho a^3}(\omega_0 + 2E_\parallel + \delta E_\perp)$$

$$\times (\sigma - \sigma^*), \qquad (22)$$

$$\frac{\partial E_\parallel}{\partial z} + \frac{1}{a}\frac{\partial E_\parallel}{\partial t} = i\frac{nA}{8\,\hbar\,\rho a^3}[\delta(\omega_0 + 2E_\parallel + \delta E_\perp) + d^2 E_\perp](\sigma - \sigma^*). \qquad (23)$$

Here

$$W = \frac{1 + W_2}{4} - \rho_{22}, \quad \sigma = \sqrt{2}\rho_{21},$$

$$E_\perp = \frac{G_\perp}{\hbar}\cos 2\alpha \mathcal{E}_{xz} + \frac{G_\parallel}{2\hbar}\sin 2\alpha \mathcal{E}_{zz},$$

$$E_\parallel = \frac{G_\parallel}{2\hbar}\cos 2\alpha \mathcal{E}_{zz} - \frac{G_\perp}{\hbar}\sin 2\alpha \mathcal{E}_{xz} \qquad (24)$$

and

$$A = G_\perp^2 \cos^2 2\alpha + \frac{G_\parallel^2}{2}\sin^2 2\alpha,$$

$$\delta = \frac{2G_\perp^2 - G_\parallel^2}{2A}\sin 4\alpha, \quad d = \frac{\sqrt{2}}{A}G_\perp G_\parallel.$$

The system (20)–(23) is remarkable from the point of view of its physical applications. In the case $d = \delta = 0$ and $E_\parallel = 0$, it coincides with well-known reduced Maxwell-Bloch equations (RMB) for an isotropic two-level medium [18]. If $d = 0$, then we can put $E_\parallel = -\delta E_\perp/2$, and the system appearing is nothing but the RMB equations for an anisotropic medium [19], which describe also the propagation of one-component

acoustic pulses through the deformed paramagnetic crystals [11]. At last, Eqs. (20)–(23) are equivalent to the system of material and wave equations derived in Ref. [8] under consideration of the dynamics of transverse-longitudinal acoustic pulses in paramagnets with effective spin $S = 1/2$ in the external magnetic field presence. The complete investigation of this system turned out to be awkward. The simplest stationary solution found corresponds to the case $\delta = 0$ only. It describes the propagation of the extremely short acoustic pulse that possesses no well-defined carrier frequency in a direction parallel to the magnetic field. In contrast to this, the pulses containing the higher-frequency components will be considered below.

Owing to the last remark, further simplification of the system (20)–(23) will be achieved by applying the SVE approximation. Comparing the right-hand side of Eqs. (22) and (23), we conclude that the field $E_\parallel$ should have, in the general case, a high-frequency component proportional to $E_\perp$. Taking this into account, we make use of the next representation

$$\sigma = R\exp[i\omega(t - z/a)],$$

$$E_\perp = (\Omega_\perp/2)\exp[i\omega(t - z/a)] + \text{c.c.},$$

$$E_\parallel = (\Omega_\parallel - \delta E_\perp)/2, \qquad (25)$$

where $\omega = \omega_0 - \Delta$ is the carrier frequency $(|\Delta| \ll \omega_0)$; R, $\Omega_\perp$, and $\Omega_\parallel$ are slowly varying functions of t and z in the standard sense [20]. Substituting these expressions into Eqs. (20)–(23) and disregarding the nonresonant terms give us the following equations:

$$\frac{\partial W}{\partial t} = \frac{i}{2}(\Omega_\perp^* R - \Omega_\perp R^*), \qquad (26)$$

$$\frac{\partial R}{\partial t} = i(\Delta + \Omega_\parallel)R + i\Omega_\perp W, \qquad (27)$$

$$\frac{\partial \Omega_\perp}{\partial z} + \frac{1}{a}\frac{\partial \Omega_\perp}{\partial t} = -i\beta_\perp R, \qquad (28)$$

$$\frac{\partial \Omega_\parallel}{\partial z} + \frac{1}{a}\frac{\partial \Omega_\parallel}{\partial t} = \beta_\parallel \frac{\partial W}{\partial t}, \qquad (29)$$

where

$$\beta_\perp = \frac{n\omega_0 A}{2\,\hbar\,\rho a^3}, \quad \beta_\parallel = \frac{nAd^2}{4\,\hbar\,\rho a^3}.$$

Note that we neglect the high-frequency addition to detuning Δ on the right-hand side of Eq. (27). The influence of this term on the dynamics of the pulses, whose duration is much greater than the oscillation period, is not significant because its average value over the pulse length tends to zero. In the case of the RMB equations for anisotropic media [19], this fact was established in Ref. [21].

The system (26)–(29) we finally obtain differs only by the notation from the LSWC equations that have been investigated in detail due to their importance for theoretical study of the nonlinear dynamics of two-component electromag-

netic pulses in anisotropic resonant media [12,13]. This reveals to us one more correspondence between coherent optical and acoustic phenomena, which will be used in the next section for classification of the modes of the acoustic pulse propagation through a strained paramagnetic crystal. It should be noted here that the system of LSWC equations, as well as its gauge equivalents, arises in various physical problems (see, e.g., references in Ref. [13]). These numerous applications point to the universal physical character of the LSWC system.

One of the features of Eqs. (26)–(29) is that the field variables play in them totally different roles. Namely, $\Omega_\perp$ causes quantum transitions, whereas $\Omega_\perp$ shifts dynamically the transition frequency. It is easy to see that there exists a relation between the field components:

$$\Omega_\parallel = -\frac{d^2}{4\omega_0}|\Omega_\perp|^2 + F(t-z/a). \tag{30}$$

The last term on the right-hand side generates phase modulation of R and $\Omega_\perp$ only and vanishes after an appropriate change of variables. Without loss of a generality, we put $F(t-z/a)=0$ in what follows.

At the end of this section, we find the expressions for transverse and longitudinal components of the strain field through the variables of the LSWC equations. It follows from Eqs. (24) and (25) that

$$\mathcal{E}_{xz} = \frac{\hbar G_\perp}{A}\cos 2\alpha\{\Omega_\perp\exp[i\omega(t-z/a)]+\text{c.c.}\}$$
$$-\frac{\hbar}{2G_\perp}\sin 2\alpha\Omega_\parallel, \tag{31}$$

$$\mathcal{E}_{zz} = \frac{\hbar G_\parallel}{A}\sin 2\alpha\{\Omega_\perp\exp[i\omega(t-z/a)]+\text{c.c.}\}+\frac{\hbar}{G_\parallel}\cos 2\alpha\Omega_\parallel. \tag{32}$$

These formulas display that both components of the acoustic pulse have high-frequency and zero-frequency components in the general case. Also, Eqs. (30)–(32) reveal to us an asymmetry on the polarity of the acoustic signal: the signs of the zero harmonics of its transverse and longitudinal components are determined by α and the type of external action (tension or compression) on the crystal. It is remarkable that the ratio of the amplitudes of the zero harmonic of $\mathcal{E}_{zz}$ and $\mathcal{E}_{xz}$,

$$\frac{G_\perp}{G_\parallel}\cot 2\alpha,$$

is equal to the inverse ratio of the first harmonic case. This fact can be used in measuring the constants of the spin-phonon interaction in paramagnetic crystals.

IV. MODES OF ACOUSTIC TRANSPARENCY

The solution of Eqs. (26)–(29) that describes the propagation of the transverse-longitudinal acoustic pulse is written as given (see Ref. [13]):

$$\Omega_\perp = \sqrt{M}\exp(i\Phi), \tag{33}$$

$$\Omega_\parallel = -\frac{d^2}{4\omega_0}M, \tag{34}$$

$$W = \left(1 - \frac{\tau_p^2}{2(1+\alpha^2)}M\right)W_0. \tag{35}$$

Here

$$M = \frac{8g}{\tau_p^2[g-\alpha+\sqrt{1+(g-\alpha)^2}\cosh 2\zeta]}, \tag{36}$$

$$\Phi = \frac{\beta_\perp\alpha\tau_p}{1+\alpha^2}W_0 z - \arctan\frac{\tanh\zeta}{s} + \text{const}, \tag{37}$$

$$\alpha = \Delta\tau_p, \quad W_0 = \frac{1-3W_2}{4},$$

$$g = \frac{2\omega_0\tau_p}{d^2}, \quad \zeta = \tau_p^{-1}\left(t-\frac{z}{v_g}\right),$$

$$s = g - \alpha + \sqrt{1+(g-\alpha)^2},$$

$$v_g = a\left(1 - \frac{a\beta_\perp\tau_p^2}{1+\alpha^2}W_0\right)^{-1}. \tag{38}$$

The free parameters of the pulse presented are Δ and τ_p. For the sake of convenience we suppose hereinafter that $\tau_p > 0$.

As follows from Eqs. (33) and (37), the component $\Omega_\perp$ has the phase modulation, which leads to a local nonlinear chirping of the carrier frequency, $\omega \to \omega_{loc} = \omega + \delta\omega_{non}$, where

$$\delta\omega_{non} \equiv \frac{\partial\Phi}{\partial t} = \frac{\Omega_\parallel}{4}. \tag{39}$$

Taking into account the dynamic shift of the transition frequency, $\omega_0 \to \omega_0^{ef} = \omega_0 + \Omega_\parallel$ [see Eq. (27)], we come to the following expression for the effective detuning Δ_{ef} of the component $\Omega_\perp$ from the resonance:

$$\Delta_{ef} \equiv \omega_0^{ef} - \omega_{loc} = \Delta + \frac{3}{4}\Omega_\parallel. \tag{40}$$

Defining the pulse length T_p as the double deviation from the zero point of $t-z/v_g$, at which $|\Omega_\perp|$ is half its maximum value, we obtain, from formula (33),

$$T_p = \tau_p\text{arccosh}\left(4 + 3\frac{g-\alpha}{\sqrt{1+(g-\alpha)^2}}\right). \tag{41}$$

In the SVE approximation, both the pulse length and nonlinear shift of the carrier frequency must obviously satisfy the conditions $\omega_0 T_p \gg 1$ and $|\delta\omega_{non}| \ll \omega_0$. It can easily be shown that these inequalities are valid if $\omega_0\tau_p \gg 1$ and $\omega_0\tau_p \gg \alpha-g$. The last condition is necessary in the case $\alpha-g \gg 1$, evidently, and can be fulfilled only when $d^2 \gg 1$.

Considering the limit $\tau_p \to \infty$ in formulas (33)–(38) gives us a rationally decreasing pulse solution:

$$\Omega_\perp = \frac{8i\omega_0\kappa}{d^2(1+i\kappa^2\xi)}\exp(i\beta_\perp W_0 z/\Delta), \qquad (42)$$

$$W = \left(1 - \frac{8\kappa^2}{(1+\kappa^2)^2(1+\kappa^4\xi^2)}\right)W_0, \qquad (43)$$

where

$$\xi = \frac{4\omega_0}{d^2}\left(t - \frac{z}{v_r}\right), \quad \kappa = \sqrt{\frac{d^2}{2\omega_0}\Delta - 1},$$

$$v_r = a\left(1 - \frac{a\beta_\perp}{\Delta^2}W_0\right)^{-1}.$$

The parameter κ in Eqs. (42) and (43) is supposed to be real. This imposes a constraint on the detuning Δ of the rationally decreasing pulses: $\Delta > \Delta_r$, where $\Delta_r = 2\omega_0/d^2$.

Now, we use the expressions presented above to identify the modes of the propagation of transverse-longitudinal acoustic pulses in deformed paramagnetic crystal. These modes resemble the ones studied in Ref. [13] for the case of two-component electromagnetic pulses. In what follows, it is assumed for the sake of concreteness that the paramagnetic impurities are in thermodynamic equilibrium prior to the pulse passage ($-1/2 < W_0 < 0$).

Let us begin with the modes in which the pulse excites the paramagnetic impurities strongly. Since $W^2 + |R|^2$ is independent of t, the strongest degree of excitation happens in the case when the value of the variable W in the pulse center differs by the sign from its value at the absence of the pulse. By virtue of Eqs. (35) and (36), this condition can be formulated as

$$\tau_p^2|\Omega_\perp(\zeta = 0)|^2 = 4(1 + \alpha^2).$$

Performing some simple algebra, we put it into the next form

$$(\alpha^2 - 2g\alpha + 1)^2 = 0,$$

which yields

$$g = \frac{1}{2}\left(\alpha + \frac{1}{\alpha}\right). \qquad (44)$$

This condition can take place if $\alpha > 0$ only. It can also be rewritten as a relation between the pulse detuning Δ and parameter τ_p:
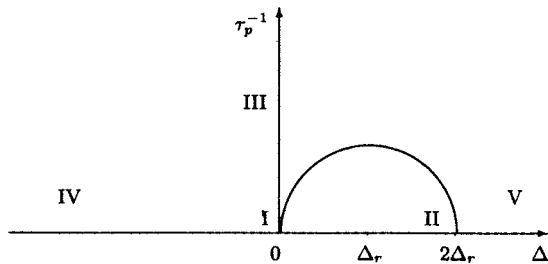

FIG. 3. Curve of the strong excitation. Domains of the pulse parameters corresponding to different modes of acoustic transparency: (I) ASIT, (II) ASIST, (III) AEOT, (IV) ANNT, and (V) APNT.

$$\tau_p^{-1} = \sqrt{\frac{4\omega_0}{d^2}\Delta - \Delta^2}. \qquad (45)$$

Setting the right-hand side to zero, we find the interval of admissible values of the detuning: $0 < \Delta < \Delta_m = 2\Delta_r$. The curve of the strong excitation (45) is plotted in Fig. 3.

It is seen from Eq. (43) that the strongest excitation in the case of a rationally decreasing pulse takes place if $\kappa = 1$. Obviously, this agrees with Eq. (45).

Substitution of relation (44) into Eqs. (33)–(38) yields

$$|\Omega_\perp| = \frac{2\sqrt{1+\alpha^2}}{\tau_p\sqrt{1+(1+\alpha^2)\sinh^2\zeta}},$$

$$\Omega_\parallel = -\frac{4\Delta}{1+(1+\alpha^2)\sinh^2\zeta},$$

$$W = \left(1 - \frac{2}{1+(1+\alpha^2)\sinh^2\zeta}\right)W_0,$$

$$v_g = a\left(1 - \frac{a\beta_\perp\tau_p^2}{1+\alpha^2}W_0\right)^{-1}. \qquad (46)$$

The corresponding pulse length is

$$T_p = 2\tau_p\,\text{arcsinh}\sqrt{\frac{3}{1+\alpha^2}}. \qquad (47)$$

According to Eqs. (40) and (46), the effective detuning of $\Omega_\perp$ from the resonance equals Δ at the edges of the pulse and -2Δ at its center. Thus, we can say that this component is resonant with quantum transitions on average over the pulse length. As a result, the largest possible change in the population spin sublevels is achieved.

If the detuning is very small ($\alpha \ll 1, g \gg 1$), then Eqs. (46) can be represented as

$$|\Omega_\perp| = \frac{2}{\tau_p}\text{sech}\zeta, \quad \Omega_\parallel = -4\Delta\,\text{sech}^2\,\zeta,$$

$$W = (1 - 2\,\text{sech}^2\,\zeta)W_0,$$

$$v_g = a(1 - a\beta_\perp\tau_p^2 W_0)^{-1}. \qquad (48)$$

One can see that $|\Omega_\parallel| \ll |\Omega_\perp|$ and the phase-modulation depth for the component $\Omega_\perp$ is much smaller than its input spectral width: $|\delta\omega_{non}| \ll 1/\tau_p$. In the exact resonance case ($\alpha = 0$, $g \to \infty$), these relations give us well-known expressions of the ASIT theory [3–5].

Formulas (46) and (39) show that an increase of detuning (and therefore α) leads to larger amplitudes of the pulse components and to a deeper chirping of $\Omega_\perp$ toward lower frequencies. Since the pulse length decreases, the profiles of both components become sharper.

Also, it is necessary to note that the group velocity approaches the linear velocity a of acoustic waves as detuning increases. Nevertheless, a strong excitation of the paramagnetic impurities takes place: the largest possible change in the level population is reached at the center of the pulse. This

is evidently explained by the fact that a growth in the amplitude of the pulse increases its power. The ensuing higher rate of excitation and deexcitation processes leads to a higher soliton propagation velocity.

It is clear from Eq. (44) that each particular value of g corresponds to two distinct values of α. We associate the domain of relatively small detuning ($\alpha < 1$) with the ASIT mode, since it is implemented in the case $\alpha = 0$. When $\alpha > 1$, we say that the pulse solutions given by Eqs. (46) propagate in the mode of acoustic self-induced supertransparency (ASIST), thus emphasizing the fact that the group velocity decrease is lower as compared to the ASIT mode while excitation is equally strong. Since $|\Delta_{ef}| \ll \omega_0$ in the SVE approximation, the ASIST mode must be most strongly manifested if $d^2 \gg 1$. The domains of the values of the pulse parameters, which correspond to these modes, are schematized in Fig. 3.

When $g \gg 1$ and $\alpha \gg 1$, the amplitudes of the components $\Omega_\perp$ and $\Omega_\parallel$ of the ASIST pulse and the phase-modulation depth for $\Omega_\perp$ reach their limits $2\Delta_m$, $4\Delta_m$, and Δ_m, respectively. In this case, Eq. (47) gives us the following estimate for the pulse length:

$$T_p \approx \frac{2\sqrt{3}\,\tau_p}{\alpha} \approx \frac{d^2}{\omega_0}.$$

This time scale corresponds to the time scale of phase-modulation localization. A deeper phase modulation combined with a decrease in the corresponding localization time scale and a shorter pulse length can be interpreted as an effect of the spectral supercontinuum generation. Indeed, the Fourier transform of Eqs. (33) and (34) defined as

$$F_{\perp,\parallel}(\nu) = \int_{-\infty}^{\infty} e^{i\nu t}\Omega_{\perp,\parallel}\,dt$$

yields

$$|F_\perp(\nu)| = 2\pi \frac{\sqrt{g}}{\sqrt[4]{1+(g-\alpha)^2}} \frac{e^{\theta\tau_p\nu/2}}{\cosh(\pi\tau_p\nu/2)},$$

$$|F_\parallel(\nu)| = 4\pi \frac{\sinh(\theta\tau_p\nu/2)}{\sinh(\pi\tau_p\nu/2)},$$

where $\theta = \operatorname{arccot}(g-\alpha)$ $(0 < \theta < \pi)$. The absolute values of the Fourier transforms of the components $\Omega_\parallel$ and $\Omega_\perp$ reach their maximum values at $\nu = 0$ and $\nu = \nu_0$, respectively, where

$$\nu_0 = \frac{2}{\pi\tau_p} \ln\frac{\pi+\theta}{\pi-\theta}.$$

If $\alpha - g \gg 1$ or, equivalently, $\theta \to \pi$, then the spectral width of the pulse is $\delta\omega \sim 1/(\pi-\theta)\tau_p$ and the maximum of the spectral energy distribution of the component $\Omega_\perp$ is reached at frequency $\omega - \nu_0$, which is much lower than the carrier frequency ω. In this case, despite a large linear detuning from resonance ($\tau_p\Delta \gg 1$), a substantial nonlinear spectral broadening ($\tau_p\delta\omega \gg 1$) and dynamical shift in the transition frequency $\omega_0 \to \omega_0^{ef}$ lead to a generation of the resonant Fourier

components (photons), which stimulate the quantum transitions.

Let us now consider the modes of the acoustic pulse propagation in the case $g \ll 1$. Although the paramagnetic impurities are excited weakly under this condition, there exist remarkable features in their interaction with the acoustic pulses. Since $\omega_0\tau_p \gg 1$ in the SVE approximation, the value of $|d|$ must be sufficiently large to ensure that $\omega_0\tau_p/d^2 \ll 1$. This can be fulfilled for some values of α if $|G_\perp| \gg |G_\parallel|$ or vice versa.

First, we assume that the detuning from resonance is large ($|\alpha| \gg 1$). If $\alpha < 0$ (or $\omega > \omega_0$), then Eqs. (33)–(38) give

$$|\Omega_\perp| = \frac{2}{\tau_p}\sqrt{\frac{g}{|\alpha|}}\operatorname{sech}\zeta,$$

$$\Omega_\parallel = -\frac{2}{|\alpha|\tau_p}\operatorname{sech}^2\zeta,$$

$$W = \left(1 - \frac{2g}{|\alpha|^3}\operatorname{sech}^2\zeta\right)W_0,$$

$$v_g = v_r. \tag{49}$$

Comparing these expressions with Eqs. (48), we see that the amplitude of $\Omega_\perp$ is much smaller than in the ASIT mode, whereas the respective $\Omega_\parallel$'s are comparable. We also note that

$$\frac{|\Omega_\perp(\zeta=0)|}{|\Omega_\parallel(\zeta=0)|} = \sqrt{g|\alpha|}:$$

i.e., the amplitude ratio can have an arbitrary value. The paramagnetic impurities remain almost unexcited as the soliton described by Eq. (49) propagates through the crystal, and the soliton velocity decreases only very slightly. The phase-modulation depth is also small ($|\delta\omega_{non}| \ll 1/\tau_p$), and the effective detuning Δ_{ef} only increases as the component $\Omega_\parallel$ is generated. This leads to an even weaker excitation of the impurities as compared to that induced by the input pulse.

Now, let us suppose that $g \ll 1$ and $\alpha \gg 1$. In this case Eqs. (33)–(38) imply

$$|\Omega_\perp| = \frac{4\sqrt{g\alpha}}{\tau_p\sqrt{1+4\alpha^2\sinh^2\zeta}},$$

$$\Omega_\parallel = -\frac{8\Delta}{1+4\alpha^2\sinh^2\zeta},$$

$$W = \left(1 - \frac{8g}{\alpha(1+4\alpha^2\sinh^2\zeta)}\right)W_0,$$

$$v_g = v_r. \tag{50}$$

Note that expressions (46) and (50) are somewhat similar: in both cases, the solitons are sharply peaked and their propagation velocities are nearly equal to a. However, these modes are essentially different in terms of the behavior of the paramagnetic impurities. Whereas they are strongly excited

as the soliton described by Eqs. (46) propagates through the crystal, no significant excitation is caused in the case of Eqs. (50). Indeed, since the effective detuning Δ_{ef} of the pulse described by Eqs. (50) is -5Δ at its center, it is not resonant with the paramagnetic impurities on average over the pulse length. However, according to Eqs. (49) and (50), the excitation of the paramagnetic impurities at $\omega < \omega_0$, being relatively weak, is still stronger than that at $\omega > \omega_0$. The reason is that the effective detuning decreases toward the pulse center when $\omega < \omega_0$, owing to the component $\Omega_\parallel$, and increases when $\omega > \omega_0$. Thus, a comparison of Eqs. (49) with (50) demonstrates obvious asymmetry with respect to the detuning of $\Omega_\perp$. Since $\alpha < 0$ for the pulses described by Eqs. (49) and $\alpha > 0$ for ones described by Eqs. (50), we refer to the corresponding modes as acoustic negative and positive nonresonant transparency (ANNT and APNT), respectively.

If $g \ll 1$ and $|\alpha| \ll 1$ (detuning is small), then Eqs. (33)–(38) lead to expressions identical to those found in Ref. [12]:

$$|\Omega_\perp| = \frac{2}{\tau_p}\sqrt{2g}\,\mathrm{sech}^{1/2}2\zeta,$$

$$\Omega_\parallel = -\frac{4}{\tau_p}\,\mathrm{sech}\,2\zeta,$$

$$W = (1 - 4g\,\mathrm{sech}\,2\zeta)W_0. \tag{51}$$

The expression for group velocity coincides with one given in Eqs. (48) corresponding to the ASIT mode.

Here, we have $|\Omega_\parallel/\Omega_\perp|^2 \gg 1$ in the center of the pulse. The paramagnetic impurities are not excited, since the effective detuning is large, $3/\tau_p$. However, the propagation velocity decreases as in the case of strong excitation at $\alpha \sim 1$. Such a deceleration of the pulse is explained by the dispersion properties of a medium within the higher-frequency component bandwidth. According to [12], where this effect was studied in detail for the optical solitons, we call the mode considered an acoustic extraordinary transparency (AEOT). The roles of the ordinary and extraordinary pulse components are played here by $\Omega_\perp$ and $\Omega_\parallel$, respectively. The existence of this mode for transverse-longitudinal acoustic pulses has been revealed also in the case when the splitting of the spin sublevels is produced by the external magnetic field [7,10]. The domains of existence of the modes with $g \ll 1$ are presented in Fig. 3.

It should be noted that expressions (33)–(38) do not change their form in the presence of detuning between $a_\parallel$ and $a_\perp$. The influence of this detuning is inessential if the condition

$$\varepsilon \equiv \left(\frac{1}{a_\perp} - \frac{1}{a_\parallel}\right)\left(\frac{1}{v_g} - \frac{1}{a}\right)^{-1} \ll 1$$

is valid. Substituting the definitions for $\beta_\perp$ and A into Eqs. (38), we obtain

TABLE I. Characteristics of the modes of acoustic transparency.

Mode	$\widetilde{\Delta}_{ef}$[a]	$|\alpha|$	g	v_g	$|\Omega_\perp/\Omega_\parallel|$	Excitation
ASIT	-2Δ	$\ll 1$	$\gg 1$	v_{ASIT}	$\gg 1$	Strong
ASIST	-2Δ	$\gg 1$	$\gg 1$	$<a$	~ 1	Strong
ANNT	$\approx\Delta$	$\gg 1$	$\ll 1$	a	Arbitrary	Weak
APNT	-5Δ	$\gg 1$	$\ll 1$	a	$\ll 1$	Weak
AEOT	$\gg|\Delta|$	$\ll 1$	$\ll 1$	$>v_{ASIT}$	$\ll 1$	Weak

[a]Note that $\widetilde{\Delta}_{ef}$ is the effective detuning of the high-frequency components at the center of the acoustic pulse; $\widetilde{\Delta}_{ef}$ is compared with Δ in the AEOT mode of absolute value.

$$\varepsilon \sim \frac{1}{a\beta_\perp\tau_p^2}\left(\frac{1}{a_\perp} - \frac{1}{a_\parallel}\right) \sim \frac{\hbar\rho a_\parallel^2}{nG_\parallel^2\omega_0\tau_p^2}\left(\frac{a_\parallel}{a_\perp} - 1\right).$$

Taking for Fe^{2+}:MgO [3,6,14,22,23] $n \sim 10^{17}$ cm^{-3}, $\omega_0 \sim 10^{10}$ s^{-1}, $G_\parallel \sim 10^{-13}$ erg, $\rho \approx 2$ g/cm^3, $a_\parallel \sim 5 \times^5$ cm/s, $a_\parallel/a_\perp \sim 1.5$, and $\tau_p \sim 10^{-8}$ s, we find $\varepsilon \sim 0.1$. This estimation shows that we can neglect detuning between the linear velocities of transverse and longitudinal acoustic waves for typical values of the ASIT parameters.

V. CONCLUSION

In this paper, we have investigated the soliton modes of the acoustic transparency in a strained cubic crystal containing resonant paramagnetic impurities with effective spin $S = 1$. It is supposed that the linear velocities of transverse and longitudinal sound are close and the pulses propagate through the crystal under arbitrary angle with respect to the direction of the external deformation parallel to the fourth-order symmetry axis.

We have allocated five modes of acoustic transparency, which differ by the propagation velocity of the transverse-longitudinal pulses and degree of excitation of the paramagnetic impurities (see Table I). The acoustic self-induced transparency is characterized by strong excitation and substantial deceleration in the pulse propagation velocity relative to linear velocities. Self-induced supertransparency differs from ASIT in that the decrease of velocity is small, but the paramagnetic impurities are strongly excited as well. The solitons of ASIST have larger amplitudes and smaller lengths as compared to ASIT solitons, and their high-frequency spectral components are strongly modulated. The carrier frequency of the transverse and longitudinal components in this mode is lower than the resonant frequency. The modes, in which the trapping of the populations of the spin sublevels takes place, are also identified. The pulses propagating in the acoustic extraordinary transparency mode are characterized by small detuning of the high-frequency components and dominant role of the zero-frequency ones. Their group velocity substantially changes and may become comparable to that of pulses in ASIT and ASIST modes. In the acoustic positive and negative nonresonant transparency modes, the pulse velocity changes insignificantly and the absolute value of detuning is large. The most substantial difference between

these modes concerns the behavior of the effective detuning of the high-frequency components. In the ANNT mode, it remains virtually constant. If a pulse propagates in the APNT mode, then the effective detuning changes sign due to the influence of the zero-frequency component. Since the local frequency passes through a resonance, a slightly stronger excitation occurs in this case and the pulses are sharply peaked, as in the ASIST mode.

In this study, we ignored inhomogeneous broadening of the spin sublevels. An investigation allowing for this effect may throw light on the pulse area theorem, providing a basis for analysis of the stability of the transverse-longitudinal acoustic pulses. Also, it would be interesting to identify the distinctive features of the dynamics of picosecond transverse-longitudinal acoustic pulses in a system of effective spins $S=1$. Unlike the present consideration, the SVE approximation is inapplicable to this case.

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